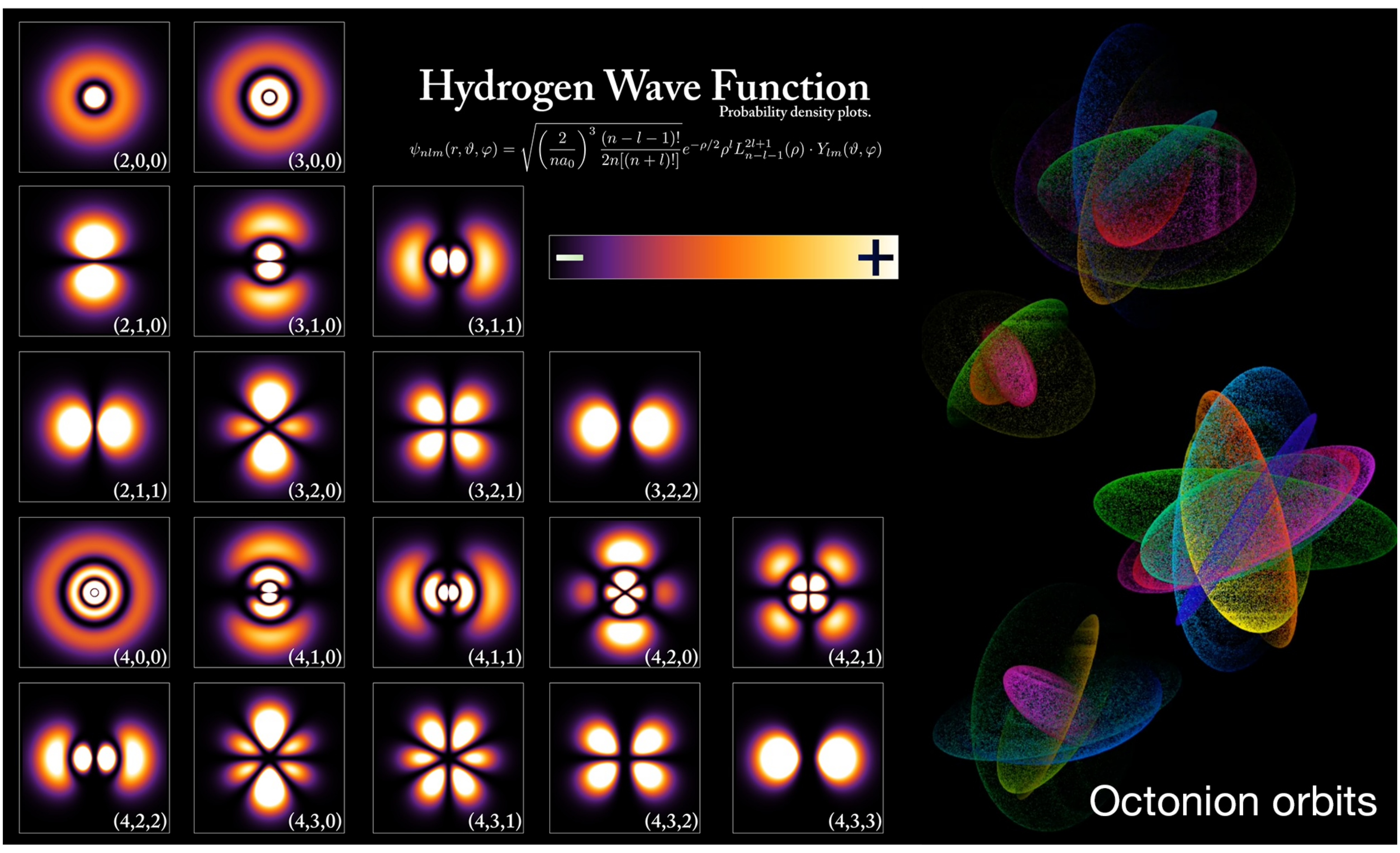

QUANTUM MECHANICS FROM PILOT WAVE THEORY

Quantum mechanics from pilot wave theory

Pilot wave theory fully (and deterministically) captures quantum mechanics, and it does so with elegance and ease. In fact, when we assume that particles (photons, electrons, etc.) are point-like entities that follow continuous and causally defined trajectories with well-defined positions $\xi(t)$, and that every particle is surrounded by a physically real wave field (pilot wave) $\psi(\boldsymbol{r},t)$ that guides it, we only need three supplementary conditions to perfectly choreograph all of quantum mechanics. Those conditions are:

1. The wave $\psi(\boldsymbol{r},t)$ evolves according to the Schrödinger equation,
2. The probability distribution of an ensemble of particles described by the wave function ψ, is $P=|\psi|^2$, and
3. Particles are carried by their local "fluid" flow. In other words, the change of particle's position with respect to time is equal to the local stream velocity $d\xi(t)/dt=\boldsymbol{v}$, where $\boldsymbol{v}=\nabla S/m$, and the "velocity potential" S is related to the phase φ of ψ by $S(\boldsymbol{r},t)=\hbar\varphi(\boldsymbol{r},t)$.

From here, obtaining a full hydrodynamic account of quantum mechanics is simply a matter of expressing the evolution of the system in terms of its fluid properties: the fluid density $\rho(\boldsymbol{r})$, the velocity potential S, and stream velocity $\boldsymbol{v}$.

The first step is to write down the Schrödinger equation in its hydrodynamic form[21]

$$\psi=\sqrt{\rho}\,e^{\frac{iS}{\hbar}}$$

Then we express fluid conservation via the continuity equation, which states that any change in the amount of fluid in any volume must be equal to rate of change of fluid flowing into or out of the volume—no fluid magically appears or disappears:

$$\tfrac{\partial\rho}{\partial t}+div(\rho\,\tfrac{\nabla S}{m})=0$$

From this it follows (given that particles are carried by their guiding waves) that the path of any particle is determined by the evolution of the velocity potential $\partial S/\partial t$, which is:

$$\tfrac{\partial S}{\partial t}=-\tfrac{1}{2m}(\nabla S)^2-V-Q$$

This evolution depends on both the classical potential V and the "quantum potential" Q, where: [22]

$$Q=-\tfrac{\hbar^2}{2m}\tfrac{\nabla^2\sqrt{\rho}}{\sqrt{\rho}}=-\tfrac{\hbar^2}{4m}[\tfrac{\nabla^2\rho}{\rho}-\tfrac{1}{2}(\tfrac{\nabla\rho}{\rho})^2]$$

That's it. We now have a hydrodynamic model that fully reproduces the behavior of quantum particles in terms of a potential flow.

Note that, from a classical or realist perspective, the assumptions held by this formalism are far less alarming than those maintained in canonical quantum mechanics (which regards the wave function to be an ontologically vague element of Nature, inserts an ad hoc time-asymmetric process into Nature—wave function collapse, abandons realism and determinism, etc.). Nevertheless, being based on an approximation of the more natural ontology, the auxiliary assumptions of this construction still cry out for a more complete understanding. So let's address them.

> Condition 1: The wave $\psi(\boldsymbol{r},t)$ evolves according to the Schrödinger equation.

Every physical medium has a wave equation that details how waves mechanically move through it. Under de Broglie's original assumption that pilot waves are mechanically supported by a physical sub-quantum medium, the idea that the pilot wave $\psi(\boldsymbol{r},t)$ evolves according to the Schrödinger equation is completely natural—so long as the fluid has the right properties (e.g. behaves like a superfluid). But the de Broglie-Bohm theory doesn't explicitly assume a physical medium. [23] As a consequence, it must tack on the assumption that the pilot wave (whatever it is a wave of) evolves (for some reason) according to the Schrödinger equation.

It's worth pointing out that the Schrödinger equation was originally derived to elucidate how photons move through the aether—the medium evoked to explain how light is mechanically transmitted. The aether was considered to be a "perfect fluid", which meant that it had zero viscosity. When the aether fell out of fashion the medium was dropped but the wave equation remained, leaving an open-ended question about what light was waving through.

When we fail to stipulate a physical medium, evolution according to the Schrödinger equation becomes a necessary additional (brute) assumption. With the physical medium in place (especially one with zero viscosity) the wave equation immediately and naturally follows as a descriptor of how waves mechanically move through that medium.

> Condition 2: The probability distribution of an ensemble of particles described by the wave function ψ, is $P=|\psi|^2$.

In order to establish that the equilibrium relation $P=|\psi|^2$ is a natural expectation for arbitrary quantum motion, Bohm and Vigier proposed a hydrodynamic model infused with a special kind of irregular fluctuations. [24] To explain those fluctuations, they pointed out that the equations governing the ψ field could "have nonlinearities, unimportant at the level where the theory has thus far been successfully applied, but perhaps important in connection with processes involving very short distances. Such nonlinearities could produce, in addition to many other qualitatively new effects, the possibility of irregular turbulent motion." [25]

Bohm and Vigier went on to note that if photons and particles of matter have a granular substructure, analogous to the molecular structure underlying ordinary fluids, then the irregular fluctuations are merely random fluctuations about the mean (potential) flow of that fluid. They went on to prove that with these fluctuations present, an arbitrary probability density will always decay to $|\psi|^2$—its equilibrium state. This proof was extended to the Dirac equation and the many-particle problem. [26]

In short, in order to justify the equilibrium relation, Bohm and Vigier returned to de Broglie's original idea—that particles are intersecting (non-linear) waves in a sub-quantum fluid surrounded by a (linear) pilot wave. The substructure of that fluid, how its inner parts mix and move about, is naturally responsible for the fluctuations that create the equilibrium relation—in perfect analogy to how Brownian motion is caused by the collisions and rearranging of molecules in the fluid it is in.

Without assuming the physical existence of this sub-quantum fluid, the wave equation and the equilibrium relation are mysterious and unexpected conditions—additional brute assumptions. With the fluid, they naturally follow.

> Condition 3: The change of particle's position with respect to time is equal to the local stream velocity $d\xi(t)/dt=\boldsymbol{v}$, where $\boldsymbol{v}=\nabla S/m$, and the "velocity potential" S is related to the phase φ of ψ by $S(\boldsymbol{r},t)=\hbar\varphi(\boldsymbol{r},t)$.

Relating the velocity potential S to the phase φ of ψ by $S(\boldsymbol{r},t)=\hbar\varphi(\boldsymbol{r},t)$, means that the phases of both (the pulsing particle and the surrounding wave) coincide. This condition—that "the particle beats in phase and coherently with its pilot wave"—is known as de Broglie's "guiding" principle. It "ensures that the energy exchange (and thus coupling) between the particle and its pilot wave is most efficient," [27] and that the core of the particle is carried along with the linear wave ψ.

Given that what de Broglie really had in mind was that particles were intersecting waves in some fluid (pulsating non-linear waves), and that pilot waves were the linear extensions of those waves into the rest of the fluid, this condition may feel completely natural—automatically imported. But the simplified model doesn't have that advantage. That is, under the approximation that particles are point-like structureless entities, it becomes necessary to additionally assert that (for some reason) those particles possess a phase, which pulses in sync with the surrounding pilot wave. This condition secures that the velocity of the particle matches the local stream velocity of the fluid.

The moral of this story is that all of the auxiliary premises in the de Broglie-Bohm theory are necessitated by the model's omission of the sub-quantum fluid that is responsible for the effects it is capturing—by what it washes out by way of approximation. In other words, these assumptions are consequences of the fact that the de Broglie-Bohm theory is a mean-field approximation of the real dynamics. To more viscerally connect with the quantum world, to have a richer understanding of quantum phenomenon while minimizing the number of our auxiliary assumptions, we have to tell the story from the perspective of the more complete ontology—the one that mirrors what's actually going on in Nature—the one that de Broglie originally had in mind. [28] This is the aim of quantum space theory.

[1] Pilot-wave theories (also called nonlocal hidden-variable theories) are a family of realist interpretations of quantum mechanics that believe that the statistical nature of quantum mechanics is due to an ignorance of an underlying more fundamental real dynamics, and that microscopic particles follow real trajectories just like larger classical bodies do. Quantum space theory falls under the family of models categorized as vacuum-based pilot-wave theories. It logically overlaps with both stochastic electrodynamics and superfluid vacuum theory. For a modern review of pilot wave theories, see: John W. M. Bush, "Pilot-Wave Hydrodynamics". Annu. Rev. Fluid Mech. 2015. 47:269–92 (2015).
[2] Louis de Broglie, "Interpretation of quantum mechanics by the double solution theory". Annales de la Fondation, Volume 12, no. 4 (1987).
[3] David Bohm, "A Suggested Interpretation of the Quantum Theory in Terms of 'Hidden Variables' I". *Physical Review*. **85** (2): 166–179 (1952).
[4] Stanley Jeffers, "Jean-Pierre Vigier and the Stochastic Interpretation of Quantum Mechanics" (2000).
[5] This echoes Dirac's sentiment that, "the perfect vacuum as an idealized state, not attainable in practice." The true vacuum is much richer than that trivial state and not "needs elaborate mathematics for its description." It acts like a fluid, or "an aether, subject to quantum mechanics and conforming to relativity…" Paul A. M. Dirac, *Nature* (London) 169, 702 (1952).
[6] If the vacuum were a composite of periodically arranged quanta, instead of being randomly arranged, then wave pulses passing through the vacuum would spread out (scatter and dilute). Randomness causes interference between multiple scattering paths to keep wave pulses completely localized. Anderson, P. W., "Absence of Diffusion in Certain Random Lattices". *Phys. Rev.* **109** (5): 1492–1505 (1958); Roati, Giacomo; et al., "Anderson localization of a non-interacting Bose-Einstein condensate". *Nature*. **453** (7197): 895–898 (2008).
[7] See: https://en.wikipedia.org/wiki/Quantum_vortex – Vortex-quantisation_in_a_superfluid
[8] Ross Anderson & Robert Brady, "Why quantum computing is hard—and quantum cryptography is not provably secure" (2013).
[9] Frank Wilczek. (December 29, 2011). Beautiful Losers: Kelvin's Vortex Atoms. NOVA: http://www.pbs.org/wgbh/nova/physics/blog/2011/12/beautiful-losers-kelvins-vortex-atoms/ Much of this section follows this article.
[10] Lord Kelvin (Sir William Thomson), On Vortex Atoms. Proceedings of the Royal Society of Edinburgh, Vol. VI, 1867, pp. 94-105. Reprinted in Phil. Mag. Vol. XXXIV, 1867, pp. 15-24.
[11] Thad Roberts, Einstein's Intuition: Visualizing Nature in Eleven Dimensions. (2016).
[12] Frank Wilczek. (2011, December 29). Beautiful Losers: Kelvin's Vortex Atoms. NOVA: http://www.pbs.org/wgbh/nova/physics/blog/2011/12/beautiful-losers-kelvins-vortex-atoms/ Much of this section follows this article.
[13] Albert Einstein, *Ann. Phys. (Leipzig)* **17**, 132. "Concerning an Heuristic Point of View Toward the Emission and Transformation of Light" (1905).
[14] Einstein eventually came to the viewpoint that "quantum statistics should be due to a real subquantal physical vacuum alive with fluctuations and randomness." Stanley Jeffers, "Jean-Pierre Vigier and the Stochastic Interpretation of Quantum Mechanics" (2000).
[15] Louis de Broglie, *Ann. Phys. (Paris)* **3**, 22 (1925).
[16] Louis de Broglie, "Interpretation of quantum mechanics by the double solution theory". Annales de la Fondation, Volume 12, no. 4 (1987); Stanley Jeffers, "Jean-Pierre Vigier and the Stochastic Interpretation of Quantum Mechanics" (2000).
[17] Louis de Broglie, "Interpretation of quantum mechanics by the double solution theory". Annales de la Fondation, Volume 12, no. 4 (1987).
[18] David Bohm, *Phys. Rev.* **85**, 166, 180 (1952).
[19] Stanley Jeffers, "Jean-Pierre Vigier and the Stochastic Interpretation of Quantum Mechanics" (2000).
[20] https://www.quora.com/Why-dont-more-physicists-subscribe-to-pilot-wave-theory/answer/Thad-Roberts
[21] Erwin Madelung, *Physik* **40**, 332 (1926). See also: Madelung equations.
[22] is called the Laplace operator and it represents the divergence of the gradient. Note that equations (2) and (3) do not contain the actual location of the particle , which is required to produce an exact output. That is, in order to use this theory to make an exact prediction of where the particle will end up, we must specify exactly where it was at some point. The quantum potential represents how much vacuum mixing redirects the particle, an effect that becomes less intense as the mass of the particle (the distortion magnitude of the soliton) increases.
[23] Many physicists imagine a non-physical field supporting these wave dynamics instead of a physical fluid medium.
[24] David Bohm & Jean-Pierre Vigier, *Phys. Rev.* **96**, 208 (1954).
[25] Ibid.
[26] Stanley Jeffers, "Jean-Pierre Vigier and the Stochastic Interpretation of Quantum Mechanics" (2000).
[27] Ibid.
[28] Louis de Broglie, *Une tentative d'interprétation causale et non-linéaire de la Mécanique Ondulatorie* (Gauthier-Villars, Paris), (1956).

where

- philosophy of physics
- theoretical physics and
- applied physics meet